YEAR 7 — REASONING WITH NUMBER
Prime numbers and Proof

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:
- Find and use multiples
- Identify factors of numbers and expressions
- Recognise and identify prime numbers
- Recognise square and triangular numbers
- Find common factors including HCF
- Find common multiples including LCM

Keywords

Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number.
Prime: an integer with only 2 factors.
Conjecture: a statement that might be true (based on reasoning) but is not proven.
Counterexample: a special type of example that disproves a statement.
Expression: a maths sentence with a minimum of two numbers and at least one math operation (no equals sign)
HCF: highest common factor (biggest factor two or more numbers share)
LCM: lowest common multiple (the first time the times table of two or more numbers match)

Multiples   The "times table" of a given number

All the numbers in this lists below are multiples of 3.

$3, 6, 9, 12, 15…$

$3x, 6x, 9x …$

This list continues and doesn't end

x could take any value and as the variable is a multiple of 3 the answer will also be a multiple of 3

Non example of a multiple

45 is not a multiple of 3 because it is 3 x 15

Not an integer

Factors

Arrays can help represent factors

5 x 2 or 2 x 5

Factors of 10
1, 2, 5, 10

10 x 1 or 1 x 10

The number itself is always a factor

Factors and expressions

| x | x | x | x | x | x |

$6x × 1$ OR $6 × x$

Factors of 6x
$6, x, 1, 6x, 2x, 3, 3x, 2$

x	x
x	x
x	x

$2x × 3$

| x | x | x |
| x | x | x |

$3x × 2$

Prime numbers

- Integer
- Only has 2 factors
- 1 and itself

2

The first prime number
The only even prime number

Learn or how-to quick recall…

$2, 3, 5, 7, 11, 13, 17, 19, 23, 29…$

Square and triangular numbers

Square numbers

odd   even   odd

Representations are useful to understand a square number n²

$1, 4, 9, 16, 25, 36, 49, 64 …$

Triangular numbers

Representations are useful – an extra counter is added to each new row

Add two consecutive triangular numbers and get a square number

$1, 3, 6, 10, 15, 21, 28, 36, 45…$

Common factors and HCF

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18   1, 2, 3, 6, 9, 18

30   1, 2, 3, 5, 6, 10, 15, 30

Common factors
(factors of both numbers)
1, 2, 3, 6

HCF = 6

6 is the biggest factor they share

Common multiples and LCM   Common multiples are multiples two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9   9, 18, 27, 36, 45, 54

12   12, 24, 36, 48, 60

LCM = 36   The first time their multiples match

| 9 | 18 | 27 | 36 | 45 |
| 12 | 24 | | 36 | 48 |

Comparing fractions

$\frac{3}{5}$ and $\frac{7}{10}$

Compare fractions using a LCM denominator

$\frac{6}{10}$ and $\frac{7}{10}$

Conjectures and counterexamples

Conjecture

1, 2, 4....
The numbers in the sequence are doubling each time.

A pattern that is noticed for many cases

Counterexamples

This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Product of prime factors

Multiplication part-whole models

30
2   15
5   3

30
3   10
2   5

30
5   6
2   3

All three prime factor trees represent the same decomposition

Multiplication is commutative

30 = 2 x 3 x 5

Multiplication of prime factors

Using prime factors for predictions

e.g. 60   30 x 2   2 x 3 x 5 x 2
     150   30 x 5   2 x 3 x 5 x 5