

Year 9 Maths

Scheme of work

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	Reasoning with Algebra						Constructing in 2 and 3 Dimensions					
	Straight Line Graphs		Forming and Solving Equations		Testing Conjectures		Three-dimensional Shapes			Constructions and Congruency		
Spring	Reasoning with Number						Reasoning with Geometry					
	Numbers		Using Percentages		Maths and Money		Deduction		Rotation and Translation		Pythagoras' Theorem	
Summer	Reasoning with Proportion						Representations and Revision					
	Enlargement and Similarity		Solving Ratio & Proportion Problems		Rates		Probability		Algebraic Representation	Revision		

YEAR 9
KNOWLEDGE ORGANISERS



Autumn 1: Reasoning with Algebra

1. Straight Line Graphs
2. Forming and Solving Equations
3. Testing Conjectures

Year 9

Autumn 1: Reasoning with algebra

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR
Block 1 Straight line graphs	Lines parallel to the axes , $y = x$ and $y = -x$	333
	Using tables of values	334
	Compare gradients	335
	Compare intercepts	336
	Understand and use $y = mx + c$	337
	Write an equation in the form $y = mx + c$ (H)	338
	Find the equation of a line graph	339
	Interpret gradient and intercepts of real-life graphs	340
	Model real-life graphs involving inverse proportion (H)	341
	Explore perpendicular lines (H)	342
Additional Key Vocabulary: <u>Axis, Horizontal, Vertical, Linear, Co-ordinate, Curve, Reciprocal</u>		
Block 2 Forming and solving equations	Solve one-and two-step equations and inequalities	343
	Solve one-and two-step equations and inequalities with brackets	344
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	Solve equations with unknowns on both sides	346
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	Solving equations and inequalities in context	348
	Substituting into formulae and equations	349
	Rearrange formulae (one-step)	350
	Rearrange formulae (two-step)	351
	Rearrange complex formulae including brackets and squares (H)	352
Additional Key Vocabulary: <u>Solution, Inverse operation, Coefficient, Variable, Subject</u>		
Block 3 Testing Conjectures	Factors, Multiples , and Primes	353
	True or False ?	354
	Always, Sometimes, Never true	355
	Show that	356
	Conjectures about number	357
	Expand a pair of binomials	358
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Additional Key Vocabulary: <u>Odd, Even, Common, Verify, Prove, Demonstrate, Quadratic, Factorise</u>		

YEAR 9 — REASONING WITH ALGEBRA... Straight Line Graphs

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Compare gradients
- Compare intercepts
- Understand and use $y = mx + c$
- Find the equation of a line from a graph
- Interpret gradient and intercepts of real-life graphs

Keywords

Gradient: the steepness of a line

Intercept: where two lines cross. The y-intercept: where the line meets the y-axis

Parallel: two lines that never meet with the same gradient

Co-ordinate: a set of values that show an exact position on a graph

Linear: linear graphs (straight line) — linear common difference by addition/ subtraction

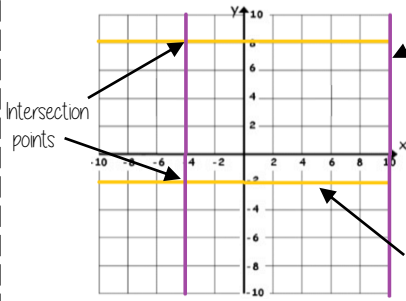
Asymptote: a straight line that a graph will never meet

Reciprocal: a pair of numbers that multiply together to give 1

Perpendicular: two lines that meet at a right angle

Lines parallel to the axes

R



All the points on this line have a x coordinate of 10

Lines parallel to the y axis take the form $x = a$ and are vertical

Lines parallel to the x axis take the form $y = a$ and are horizontal

All the points on this line have a y coordinate of -2 eg (3, -2) (7, -2) (-2, -2) all lay on this line because the y coordinate is -2

'a' can be ANY positive or negative value including 0

Plotting $y = mx + c$ graphs

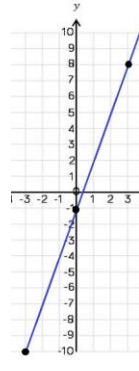
R

$y = 3x - 1$ → 3 x the x coordinate then - 1

x	-3	0	3
y	-10	-1	8

Draw a table to display this information

This represents a coordinate pair (-3, -10)



You only need two points to form a straight line

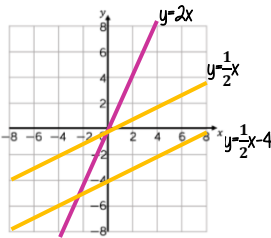
Plotting more points helps you decide if your calculations are correct (if they do make a straight line)

Remember to join the points to make a line

Compare Gradients

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line



The greater the gradient — the steeper the line

Positive gradients

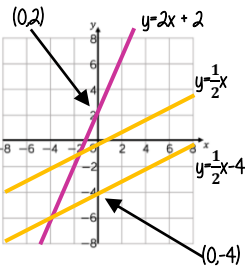
Parallel lines have the same gradient

Negative gradients

Compare Intercepts

$y = mx + c$

The value of c is the point at which the line crosses the y-axis Y intercept



The coordinate of a y intercept will always be (0,c)

Lines with the same y-intercept cross in the same place

$y = mx + c$

The coefficient of x (the number in front of x) tells us the gradient of the line

$y = mx + c$
y and x are coordinates

The value of c is the point at which the line crosses the y-axis Y intercept

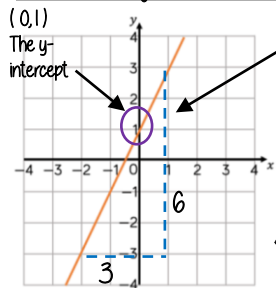
The equation of a line can be rearranged. Eg

$y = c + mx$

$c = y - mx$

Identify which coefficient you are identifying or comparing

Find the equation from a graph



The Gradient $\frac{6}{3} = 2$

$y = 2x + 1$

The direction of the line indicates a positive gradient

Positive gradients

Negative gradients

Real life graphs

A plumber charges a £25 callout fee, and then £12.50 for every hour. Complete the table of values to show the cost of hiring the plumber.

Time (h)	0	1	2	3	8
Cost (£)	£25				£125

In real life graphs like this values will always be positive because they measure distances or objects which cannot be negative.

Direct Proportion graphs

To represent direct proportion the graph must start at the origin

A box of pens costs £2.30

Complete the table of values to show the cost of buying boxes of pens.

Boxes	0	1	2	3	8
Cost (£)		£2.30			

When you have 0 pens this has 0 cost. The gradient shows the price per pen.

The y-intercept shows the minimum charge. The gradient represents the price per mile

YEAR 9 — REASONING WITH ALGEBRA...

Forming and Solving Equations

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve inequalities with negative numbers
- Solve equations with unknowns on both sides
- Solve inequalities with unknowns on both sides
- Substitute into formulae and equations
- Rearrange formulae

Keywords

Inequality: an inequality compares two values showing if one is greater than, less than or equal to another

Variable: a quantity that may change within the context of the problem

Rearrange: Change the order

Inverse operation: the operation that reverses the action

Substitute: replace a variable with a numerical value

Solve: find a numerical value that satisfies an equation

Solve equations with brackets



$$3(2x + 4) = 30$$

$$6x + 12 = 30$$

$$6x = 18$$

$$x = 3$$

$$3(2x + 4) = 30$$

Expand the brackets

$$6x + 12 = 30$$

$$-12 \quad -12$$

$$6x = 18$$

$$-6 \quad -6$$

$$x = 3$$

Form and solve inequalities



Two more than treble my number is greater than 11

Find the possible range of values

$$3x + 2 > 11$$

Solve

$$x \leftarrow -3 \leftarrow -2 \leftarrow 11$$

$$x > 3$$

Inequalities with negatives

Method 1 Make x positive first

$$2 - 3x > 17$$

$$+3x \quad +3x$$

$$2 > 17 + 3x$$

$$-17 \quad -17$$

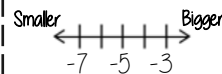
$$-15 > 3x$$

$$\div 3 \quad \div 3$$

$$-5 > x$$

x is true for any value smaller than -5

✓ CHECK IT!
 $2 - 3(-6) = 20$
TRUE/ CORRECT



Equations with unknown on both sides

$$4x + 5 = 3x + 24$$

$$-3x \quad -3x$$

$$x + 5 = 24$$

$$-5 \quad -5$$

$$x = 19$$

$$x \quad x \quad x \quad x \quad 5$$

$$x \quad x \quad x \quad 24$$

Inequalities with unknown on both sides

Solving inequalities has the same method as equations

$$5(x + 4) < 3(x + 2)$$

$$5x + 20 < 3x + 6$$

$$2x + 20 < 6$$

$$2x < -14$$

$$x < -7$$

$$5(-8 + 4) < 3(-8 + 2)$$

$$5(-4) < 3(-6)$$

$$-20 < -18$$

✓ -20 IS smaller than -18

Check it!

Method 2 Keep the negative x

$$2 - 3x > 17$$

$$-2 \quad -2$$

$$-3x > 15$$

$$\div -3 \quad \div -3$$

$$x > -5$$

x is true for any value bigger than -5

This cannot be true...

$$x < -5$$

When you multiply or divide x by a negative you need to reverse the inequality

Formulae and Equations

Substitute in values

Formulae — all expressed in symbols

Equations — include numbers and can be solved

Rearranging Formulae (one step)

$$x = y + z$$

$$x = y + z$$

Rearrange to make y the subject.

$$y = x - z$$

$$y \rightarrow +z \rightarrow x$$

$$y \leftarrow -z \leftarrow x$$

Using inverse operations or fact families will guide you through rearranging formulae

Rearranging can also be checked by substitution.

Language of rearranging...

Make XXX the subject

Change the subject

Rearrange

Rearranging Formulae (two step)

In an equation (find x)

$$4x - 3 = 9$$

$$+3 \quad +3$$

$$4x = 12$$

$$\div 4 \quad \div 4$$

$$x = 3$$

In a formula (make x the subject)

$$xy - s = a$$

$$+s \quad +s$$

$$xy = a + s$$

$$\div y \quad \div y$$

$$x = \frac{a + s}{y}$$

The steps are the same for solving and rearranging

Rearranging is often needed when using $y = mx + c$

e.g Find the gradient of the line $2y - 4x = 9$

Make y the subject first $y = \frac{4x + 9}{2}$

$$\text{Gradient} = \frac{4}{2} = 2$$

YEAR 9 — REASONING WITH ALGEBRA...

Testing conjectures

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What do I need to be able to do?

By the end of this unit you should be able to:

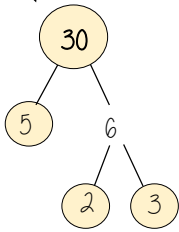
- Use factors, multiples and primes
- Reason True or False
- Reason Always, sometimes never true
- Show that reasoning
- Make conjectures about number
- Expand binomials
- Make conjectures with algebra
- Explore the 100 grid

Keywords

- Multiples:** found by multiplying any number by positive integers
- Factor:** integers that multiply together to get another number.
- Prime:** an integer with only 2 factors.
- HCF:** highest common factor (biggest factor two or more numbers share)
- LCM:** lowest common multiple (the first time the times table of two or more numbers match)
- Verify:** the process of making sure a solution is correct
- Proof:** logical mathematical arguments used to show the truth of a statement
- Binomial:** a polynomial with two terms
- Quadratic:** a polynomial with four terms (often simplified to three terms)

Factors, Multiples and Primes

Multiplication part-whole models



All three prime factor trees represent the same decomposition

HCF – Highest common factor

HCF of 18 and 30

18 1, 2, 3, 6, 9, 18

30 1, 2, 3, 5, 6, 10, 15, 30

Common factors are factors two or more numbers share

LCM – Lowest common multiple

LCM of 9 and 12

9 9, 18, 27, 36, 45, 54

12 12, 24, 36, 48, 60

Common multiples are multiples two or more numbers share



True or False?

Conjecture

A pattern that is noticed for many cases

1, 2, 4, ...
The numbers in the sequence are doubling each time.

Counterexamples



This sequence isn't doubling it is adding 2 each time

Only **one** counterexample is needed to disprove a conjecture

Always, Sometimes, Never true.

Always Every value always supports the statement

Sometimes Examples show the statement being true and counter examples to show when it is false.

Never No example supports the statement

Examples to try

- 0 and 1
- Fractions
- Negative numbers

Show that

Numerical verification

Show the stages to a solution with numerical values

Algebraic verification

Show algebraic properties of the solution
You may want to use pictorial images to support this

Proof

Simple proofs using algebra

Compare the left hand side of an equation with the right hand side – are they the same or different?

Conjectures



Even
(2n)
Multiple of 2



Odd
(2n + 1)
One more than any even

Use numerical verification first
Use pictorial verification – the representations of numbers of odd and even

Exploring the 100 square

In terms of 'n' is used to make generalisations about relationships between numbers

Positions of numbers in relation to n form expressions.

Eg one space to the right of n
 $n + 1$

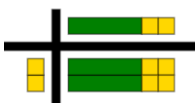
Eg One row below n
 $n + 10$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

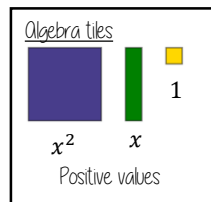
The size of the grid for generalisation changes the relationship statements

Expanding binomials

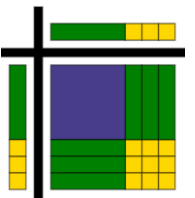
$$2(x + 2) \equiv 2x + 4$$



Algebra tiles can represent a binomial expansion
Has two terms



$$(x + 3)(x + 3) \equiv x^2 + 6x + 9$$



This is a quadratic
It has four terms which simplified to three terms

The order of the binomial has no impact on the outcome.
eg $(x + 3)(3 + x)$

YEAR 9
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**Autumn 2: Constructing in 2 and 3
Dimensions**

4. Three-dimensional Shapes
5. Constructions and
Congruency

Year 9

Autumn 2: Constructions 2D & 3D

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR
Block 4 Three-dimensional shapes	Know names of <u>2-D</u> and <u>3-D</u> shapes	361
	Recognise <u>prisms</u>	362
	Accurate <u>nets</u> of <u>cuboids</u> and other <u>3-D</u> shapes	363
	Sketch and recognise <u>nets</u> of <u>cuboids</u> and other <u>3-D</u> shapes	364
	<u>Plans</u> and <u>elevations</u>	365
	Find <u>area</u> of <u>2D</u> shapes	366
	<u>Surface area</u> of <u>cubes</u> and <u>cuboids</u>	367
	<u>Surface area</u> of <u>triangular prisms</u>	368
	<u>Surface area</u> of a <u>cylinder</u>	369
	<u>Volume</u> of <u>cubes</u> and <u>cuboids</u>	370
	<u>Volume</u> of other <u>3-D</u> shapes - <u>prisms</u> and <u>cylinders</u>	371
	Explore <u>volumes</u> of <u>cones</u> , <u>pyramids</u> , and <u>spheres (H)</u>	372
Additional Key Vocabulary: <u>Dimensions</u>, <u>Face</u>, <u>Edge</u>, <u>Vertex</u>, <u>Polygon</u>, <u>Cross-section</u>, <u>Perspective</u>		
Block 5 Constructions & congruency	Draw and <u>measure angles</u>	373
	Construct and interpret <u>scale</u> drawings	374
	<u>Locus</u> of <u>distance</u> from a point	375
	<u>Locus</u> of <u>distance</u> from a <u>straight line</u> / shape	376
	<u>Locus equidistant</u> from two points	377
	Construct a <u>perpendicular</u> from a point	378
	Construct a <u>perpendicular</u> to a point	379
	<u>Locus</u> of <u>distance</u> from two lines	380
	Construct an <u>angle bisector</u>	381
	Construct triangles from given information	382
	Explore <u>congruent</u> triangles	383
	Identify <u>congruent</u> triangles	384
Additional Key Vocabulary: <u>Estimate</u>, <u>Protractor</u>, <u>Ratio</u>, <u>Multiplier</u>, <u>Conversion</u>, <u>Construction lines</u>		

YEAR 9 — CONSTRUCTING IN 2D/3D... 3D Shapes

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Name 2D & 3D shapes
- Recognise Prisms
- Sketch and recognise nets
- Draw plans and elevations
- Find areas of 2D shapes
- Find Surface area for cubes, cuboids, triangular prisms and cylinders
- Find the volume of 3D shapes

Keywords

2D: two dimensions to the shape e.g length and width

3D: three dimensions to the shape e.g length, width and height

Vertex: a point where two or more line segments meet

Edge: a line on the boundary joining two vertex

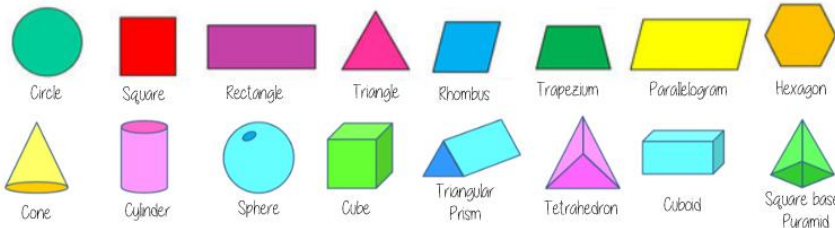
Face: a flat surface on a solid object

Cross-section: a view inside a solid shape made by cutting through it

Plan: a drawing of something when drawn from above (sometimes birds eye view)

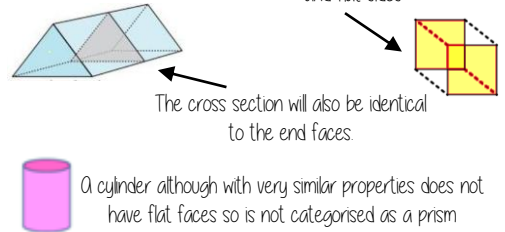
Perspective: a way to give illustration of a 3D shape when drawn on a flat surface.

Name 2D & 3D shapes

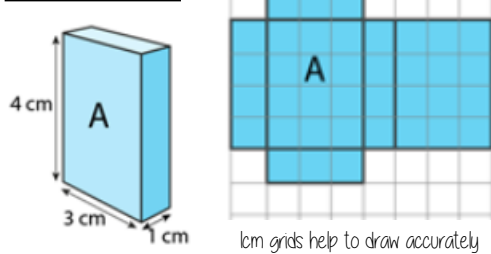


Recognise prisms

A solid object with two identical ends and flat sides

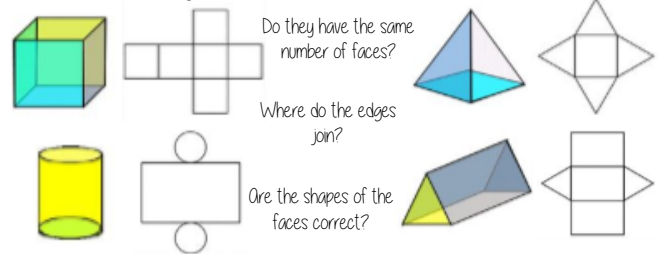


Nets of cuboids

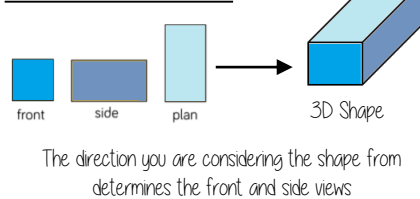


Visualise the folding of the net. Will it make the cuboid with all sides touching

Sketch and recognise nets

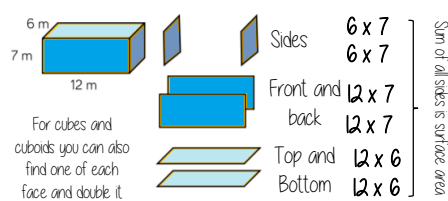


Plans and elevations

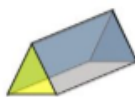


Surface area

Sketching nets first helps you visualise all the sides that will form the overall surface area



For cubes and cuboids you can also find one of each face and double it



For other shapes - not all the sides are the same, so calculate the individually

Volumes

Volume is the 3D space it takes up — also known as capacity if using liquids to fill the space



Counting cubes

Some 3D shape volumes can be calculated by counting the number of cubes that fit inside the shape

Cubes/ Cuboids = base x width x height

Remember multiplication is commutative



Cross section



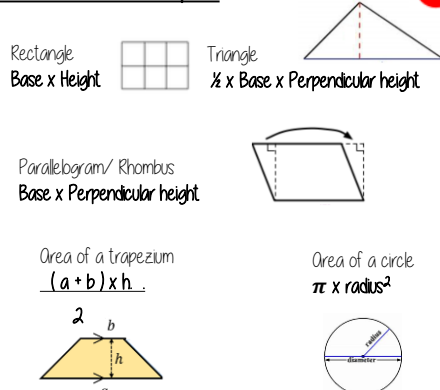
Prisms and cylinders = area cross section x height

Height can also be described as depth

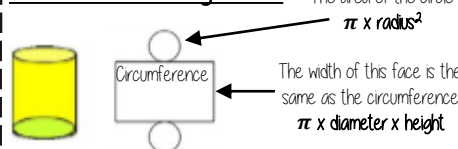
Areas — square units
Volumes — cube units

Areas and volumes can be left in terms of π

Area of 2D shapes



Surface area - cylinders



$2 \times \pi \times \text{radius}^2 + \pi \times \text{diameter} \times \text{height}$

YEAR 9 — CONSTRUCTING IN 2D/3D...

Constructions & congruency

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw and measure angles
- Construct scale drawings
- Find locus of distance from points, lines, two lines
- Construct perpendiculars from points, lines, angles
- Identify congruence
- Identify congruent triangles

Keywords

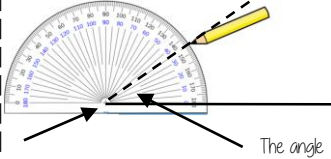
- Protractor:** piece of equipment used to measure and draw angles
- Locus:** set of points with a common property
- Equidistant:** the same distance
- Discorectangle:** (a stadium) — a rectangle with semi circles at either end
- Perpendicular:** lines that meet at 90°
- Arc:** part of a curve
- Bisector:** a line that divides something into two equal parts
- Congruent:** the same shape and size

Draw and measure angles

R

Draw a 35° angle

Make a mark at 35° with a pencil and join to the angle point (use a ruler)



Make sure the cross is at the end of the line (where you want the angle)

Scale drawings

R

A picture of a car is drawn with a scale of 1:30

For every 1cm on my image is 30cm in real life

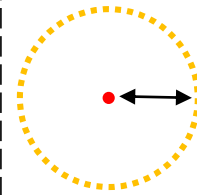
The car image is 10cm



Image: Real life
1cm : 30cm
 $\times 10$ \leftarrow 10cm : 300cm $\leftarrow \times 10$

Locus of a distance from a point

All points are equidistant (the same distance) from the fixed point in the middle

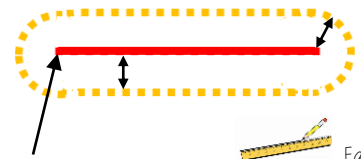


If the point is in the corner it can only make a quarter circle



Equipment needed
The radius is the distance from the fixed point

Locus of a distance from a straight line



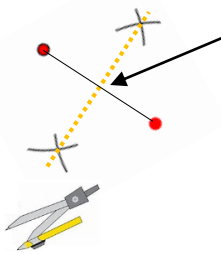
All points are equidistant (the same distance) from line

The ends of the line are fixed points



Equipment needed
The line is straight so a ruler is used for the straight lines parallel to your original line

Locus equidistant from two points

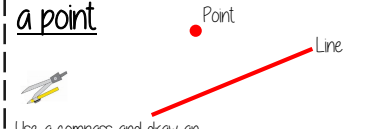


Also a perpendicular bisector
Because if the points are joined this new line intersects it at a 90°

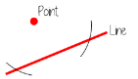


Join the intersections with a ruler.
All points on this line are equidistant from both points

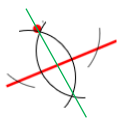
Construct a perpendicular from a point



Use a compass and draw an arc that cuts the line. Use the point to place the compass



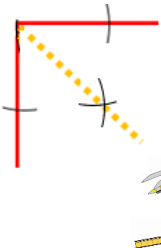
Keep the compass the same distance and now use your new points to make new intersecting arcs



Connecting the arcs makes the bisector

If P is a point on the line the steps are the same

Locus of a distance from two lines



Also an angle bisector
This cuts the angle in half

From the angle vertex draw two arcs that cut the lines forming the angle

Keep the compass the same size and use the new arcs as centres to draw intersecting arcs in the middle

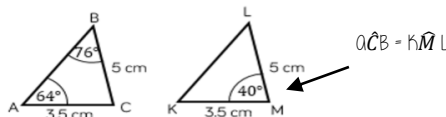
Join the vertex to the intersection

Congruent figures



Congruent figures are identical in size and shape — they can be reflections or rotations of each other

Congruent shapes are identical — all corresponding sides and angles are the same size



Because all the angles are the same and $AC=KM$ $BC=LM$ triangles ABC and KLM are congruent

Congruent triangles

Side-side-side

All three sides on the triangle are the same size

Angle-side-angle

Two angles and the side connecting them are equal in two triangles

Side-angle-side

Two sides and the angle in-between them are equal in two triangles (it will also mean the third side is the same size on both shapes)

Right angle-hypotenuse-side

The triangles both have a right angle, the hypotenuse and one side are the same

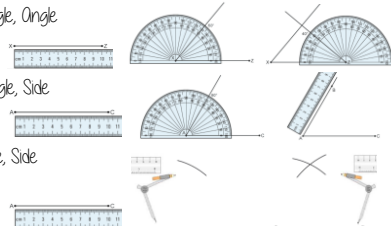
Constructing Triangles

Link to steps → R

Side, Angle, Angle

Side, Angle, Side

Side, Side, Side



YEAR 9
KNOWLEDGE ORGANISERS



Spring 1: Reasoning with Number

6. Numbers
7. Using Percentages
8. Maths and Money

Year 9

Spring 1: Reasoning with Number

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR	
Block 1 Number	<u>Integers</u> , <u>real</u> and <u>rational</u> numbers	387	
	Understand and use <u>surds</u> (H)	388	
	Work with <u>directed</u> number	389	
	Solve problems with <u>integers</u>	390	
	Solve problems with <u>decimals</u>	391	
	<u>HCF</u> and <u>LCM</u>	387	
	<u>Adding</u> and <u>subtracting fractions</u>	388	
	<u>Multiplying</u> and <u>dividing fractions</u>	389	
	Solving problems with <u>fractions</u>	390	
	Numbers in <u>standard form</u>	391	
Additional Key Vocabulary:			
Block 2 Using percentages	Use the <u>equivalence</u> of <u>fractions</u> , <u>decimals</u> , and <u>percentages</u>	397	
	Calculate <u>percentage increase</u> and <u>decrease</u>	398	
	R - Express a change as a percentage	399	
	Solve ' <u>reverse</u> ' <u>percentage</u> problems	400	
	Recognise and solve <u>percentage</u> problems (non-calculator)	401	
	Recognise and solve <u>percentage</u> problems (calculator)	402	
	Solve problems with <u>repeated percentage change (H)</u>	403	
Additional Key Vocabulary:			
Block 3 Maths and money	Solve problems with <u>bills</u> and <u>bank statements</u>	404	
	Calculate <u>simple interest</u>	405	
	Calculate <u>compound interest</u>	406	
	Solve problems with <u>Value Added Tax</u>	407	
	Calculate <u>wages</u> and <u>taxes</u>	408	
	Solve problems with <u>exchange rates</u>	409	
	Solve <u>unit pricing</u> problems	410	
Additional Key Vocabulary:			

YEAR 9 — REASONING WITH NUMBER...

Numbers

@whisto_maths

What do I need to be able to do?

- By the end of this unit you should be able to:
- Identify integers, real and rational numbers
 - Work with directed number
 - Solve problems with number
 - Find HCF/ LCM
 - Add/ Subtract fractions
 - Multiply/ Divide fractions
 - Write numbers in standard form

Keywords

- Integer:** a whole number that is positive or negative
Rational: a number that can be made by dividing two integers
Irrational: a number that cannot be made by dividing two integers
Inverse operation: the operation that reverses the action
Quotient: the result of a division
Product: the result of a multiplication
Multiples: found by multiplying any number by positive integers
Factor: integers that multiply together to get another number

Integers, real and rational numbers

Rational – root word: ratio

Real numbers: $\frac{2}{3}$ stems from 2 | ($\frac{2}{3}$ of the whole)

Irrational numbers: $\sqrt{2}$ the solution is a decimal that never ends and does not repeat

The square root of a negative is not a real number and cannot be found

HCF/LCM

1 is a common factor of all numbers

Common factors are factors two or more numbers share

HCF – Highest common factor

HCF of 18 and 30

18: 1, 2, 3, 6, 9, 18

30: 1, 2, 3, 5, 6, 10, 15, 30

HCF = 6

LCM – Lowest common multiple

LCM of 9 and 12

9: 9, 18, 27, 36, 45, 54

12: 12, 24, 36, 48, 60

LCM = 36

The first time their multiples match

Standard form

Any number between 1 and less than 10 $\rightarrow A \times 10^n$ \leftarrow Any integer

$$6 \times 10^5 + 8 \times 10^5$$

$$= 600000 + 800000$$

$$= 1400000$$

$$= 1.4 \times 10^6$$

$$(1.5 \times 10^5) \div (0.3 \times 10^3)$$

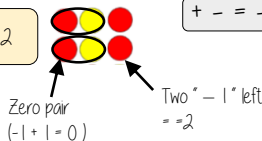
$$15 \div 0.3 \times 10^5 \div 10^3$$

$$= 5 \times 10^2$$

Directed number

Addition

$$2 + -4 = -2$$



Subtraction

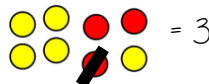
$$2 - -1 = 3$$

Representation for calculation

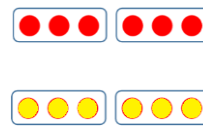
"Subtract" – means take away or remove

$$2 - -1 = 3$$

Start with the representation of 2



Multiplication



$$-2 \times -3 = 6$$

Divisions are the inverse operations

Red = -1
Yellow = 1

The act of making counters into their negative is turning them over



$$a = 5$$

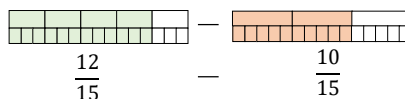
$$b = -4$$

Brackets around negative substitutions helps remove calculation errors

$$2a - b = 2 \times 5 - (-4) = 10 + 4 = 14$$

Addition/ Subtraction of fractions

$$\frac{4}{5} - \frac{2}{3}$$



$$= \frac{2}{15}$$

Use equivalent fractions to find a common multiple for both denominators

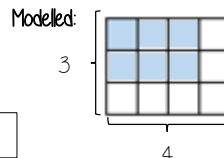
Multiplication/ Division of fractions

$$\frac{3}{4} \times \frac{2}{3}$$

This many columns

This many rows

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$$



Parts shaded

Total number of parts in the diagram

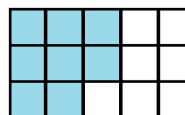
Remember to use reciprocals

$$2 \div \frac{3}{4}$$

$$2 \times \frac{4}{3}$$

Multiplying by a reciprocal gives the same outcome

Represented



$$= \frac{8}{3}$$

YEAR 9 — REASONING WITH NUMBER... Using Percentages

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use FDP equivalence
- Calculate percentage increase and decrease
- Express percentage change
- Solve reverse percentage problems
- Solve percentage problems (calculator and non calculator problems)

Keywords

- Percent:** parts per 100 — written using the % symbol
Decimal: a number in our base 10 number system. Numbers to the right of the decimal place are called decimals.
Fraction: a fraction represents how many parts of a whole value you have.
Equivalent: of equal value.
Reduce: to make smaller in value.
Growth: to increase/ to grow.
Integer: whole number, can be positive, negative or zero.
Invest: use money with the goal of it increasing in value over time (usually in a bank).
Multiplier: the number you are multiplying by.
Profit: the income take away any expenses/ costs.

FDP Equivalence

Percentage
100% = a whole = 100 hundredths

One Whole = 1

10 hundredths
10 out of 100
10%

One hundredth
(one whole split into 100 equal parts)

$$\frac{10}{100} = \frac{1}{10} = 0.10$$

ones	tenths	hundredths
	.	

Converting FDP

70/100

This also means 70 - 100

70 out of 100 squares
70 "hundredths"
= 7 "tenths"
0.7

70 hundredths = 70%

Using a calculator

Convert to a decimal

× 100 converts to a percentage

Be careful of recurring decimals
eg $\frac{1}{3} = 0.3333333$
 $\frac{1}{3} = 0.\dot{3}$
The dot above the 3

Percentage Increase/ Decrease

Decrease

100%

Decrease by 58%

42%

Increase

100%

Increase by 12%

Multiplier Less than 1

$$100 - 0.58 = 0.42$$

Multiplier More than 1

$$100\% + 12\% = 112\%$$

$$100 + 0.12 = 1.12$$

Percentage change

I bought a phone for £200
A year later sold it for £125.

100%

£200

£125

Percentage loss

All values of change compare to the ORIGINAL value

$$\frac{75}{200} \times 100 = 37.5\%$$

Reverse Percentages

40% of my number is 16
What am I thinking of?

Original Number (100%)

16

40% = 16
10% = 4
100% = 40

140% of my number is 84.
What is the original number?

Original Number (100%)

84

140% = 84
10% = 6
100% = 60

Try to scale down to 10% or 1% and then scale back up to 100%

$$\frac{\text{Difference in values}}{\text{Original value}} \times 100$$

I bought a house for £180,000, I later sold it for £216,000.

100%

£180,000

Percentage profit

Money made (profit value)

$$\frac{36000}{180000} \times 100 = 20\%$$

YEAR 9 — REASONING WITH NUMBER... Maths & Money

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with bills and bank statements
- Calculate simple interest
- Calculate compound interest
- Calculate wages and taxes
- Solve problems with exchange rates
- Solve unit pricing problems

Keywords

- Credit:** money being placed into a bank account
- Debit:** money that leaves a bank account
- Balance:** the amount of money in a bank account
- Expense:** a cost/ outgoing
- Deposit:** an initial payment (often a way of securing an item you will later pay for)
- Multiplier:** a number you are multiplying by. (Multiplier more than 1 = increasing, less than 1 = decreasing)
- Per Annum:** each year
- Currency:** the type of money a country uses.
- Unitary:** one — the cost of one.

Bills and Bank Statements

Bills — tell you the amount items cost and can show how much money you need to pay.

Some can include a total
Look for different units
(Is it in pence or pounds)

Menu	Price
Milk	89p
Tea	£1.50

Bank Statements

Bank statement can have negative balances if the money spent is higher than the money coming into the account

Date	Description	Credit	Debit	Balance
19 th Sept	Salary	£1500		£1500
19 th Sept	Mortgage		£600	£900
25 th Sept	Bday Money	£15		£915

Simple Interest

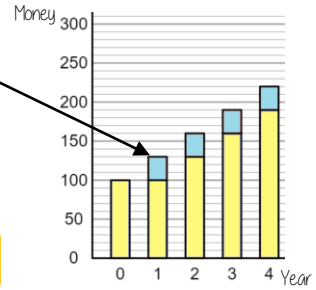
For each year of investment the interest remains the same

$$\frac{\text{Principal amount} \times \text{Interest Rate} \times \text{Years}}{100}$$

Principal amount is the amount invested in the account
e.g Invest £100 at 30% simple interest for 4 years

$$\frac{100 \times 30 \times 4}{100} = \text{£}120$$

This account earned **£120** interest.
At the end of year 4 they have **£220**



Compound Interest

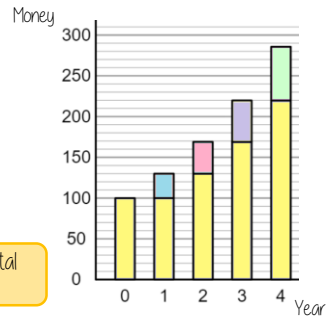
Interest is added to the current value of investment at the end of each year so the next year's interest is greater.

$$\text{Principal amount} \times \text{Multiplier}^{\text{Years}}$$

e.g Invest £100 at 30% compound interest for 4 years

$$100 \times 1.3^4 = \text{£}285.61$$

This account has **£285.61** in total at the end of the 4 years.



Value Added Tax (VAT)

VAT is payable to the government by a business. In the UK VAT is 20% and added to items that are bought.

Essential items such as food do not include VAT.

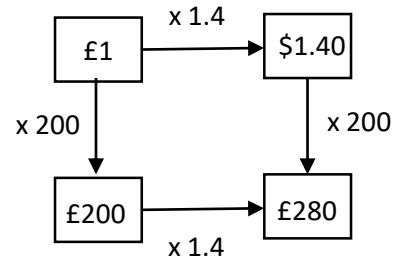
Wages and Taxes

Salaries fall into tax brackets — which means they pay this much each month from their salary.

Taxable Income	Tax Rate
£12 501 to £50 000	20%
£50 001 to £150 000	40%
over £150 000	45%

Over time:
Time and a half — means 1.5 times their hourly rate
Double — 2 times their hourly rate

Exchange Rates



When making estimates it is also useful to use estimates to check if our solution is reasonable.

Use inverse operations to reverse the exchange process

Common Currencies

United Kingdom	£	Pounds
United States of America	\$	Dollars
Europe	€	Euros

Unit Pricing

4 Oranges £1	5 cupcakes £1.20
-----------------	---------------------

$$\begin{array}{l} 4 = \text{£}1.00 \\ 2 = \text{£}0.50 \\ 1 = \text{£}0.25 \end{array} \left. \begin{array}{l} \div 2 \\ \div 2 \end{array} \right\} \begin{array}{l} 5 = \text{£}1.20 \\ 1 = \text{£}0.20 \end{array}$$

Cost per Unit

To calculate unit per cost you divide by the cost.

Cupcakes are the best value as one item has the cheapest value

There is a directly proportional relationship between the cost and number of units.

YEAR 9
KNOWLEDGE ORGANISERS



Spring 2: Reasoning with Geometry

9. Deduction

10. Rotation and Translation

11. Pythagoras' Theorem

Year 9

Spring 2: Reasoning with Geometry

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR	
Block 4 Deduction	Angles in parallel lines	411	
	Solving angles problems (using chains of reasoning)	412	
	Angles problems with algebra	413	
	Conjectures with angles	414	
	Conjectures with shapes	415	
	Link constructions and geometrical reasoning (H)	416	
Additional Key Vocabulary:			
Block 5 Rotation and translation	Identify the order of rotational symmetry of a shape	417	
	Compare and contrast rotational symmetry with line symmetry	418	
	Rotate a shape about a point on a shape	419	
	Rotate a point about a point not on a shape	420	
	Translate points and shapes by a given vector	421	
	Compare rotation and reflection of shapes	422	
	Find the result of a series of transformations (H)	423	
Additional Key Vocabulary:			
Block 6 Pythagoras Theorem	Squares and square roots	424	
	Identify the hypotenuse of a right-angled triangle	425	
	Determine whether a triangle is right-angled	426	
	Calculate the hypotenuse of a right-angled triangle	427	
	Calculate missing sides in right-angled triangles	428	
	Use Pythagoras theorem on coordinate axes	429	
	Explore proofs of Pythagoras' theorem	430	
	Use Pythagoras' theorem in 3-D shapes (H)	431	
Additional Key Vocabulary:			

YEAR 9 — REASONING WITH GEOMETRY... Deduction

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify angles in parallel lines
- Solve angle problems
- Make conjectures with angles
- Make conjectures with shapes

Keywords

- Parallel:** two straight lines that never meet with the same gradient
Perpendicular: two straight lines that meet at 90°
Transversal: a line that crosses at least two other lines
Sum: the result of adding two or more numbers
Conjecture: a statement that might be true but is not proven
Equation: a statement that says two things are equal
Polygon: a 2D shape made from straight edges
Counterexample: an example that disproves a statement

Alternate angles

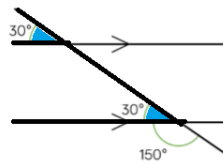
Because alternate angles are equal the highlighted angles are the same size



R

Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size

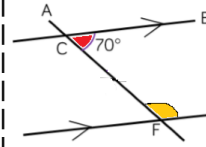


R

Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°

As angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first



R

Solving angle problems

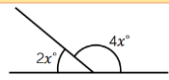
Link angle facts to algebra

Form an equation

State the reason

Solve

Angles on a straight line



$$2x + 4x = 180^\circ$$

The sum of angles on a straight line is 180°

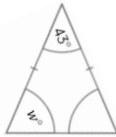
$$2x + 4x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

Vertically opposite angles
Equal

Angles around a point
 360°

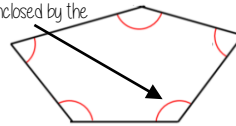


Triangles
Sum of angles is 180°

Isosceles have the same base angles

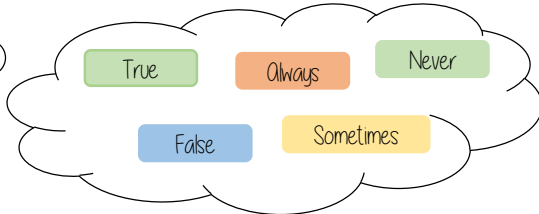
Interior Angles

The angles enclosed by the polygon



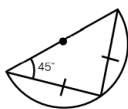
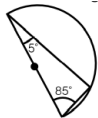
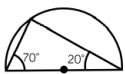
$$(\text{number of sides} - 2) \times 180$$

Making conjectures with angles



Proving a conjecture

A pattern is noticed for many cases



Apply the angle rules

Test the theory

Make conjecture

The sum of angles in a triangle is 180°

$$180 - 70 - 20 = 90$$

$$180 - 85 - 5 = 90$$

$$180 - 45 - 45 = 90$$

The angle that meets the circumference in a semi circle is 90°

Disproving a conjecture

Only one counterexample is needed to disprove a conjecture

Making conjectures with shapes

Keywords and facts to recall with shape

Area: the amount of space inside a shape
Perimeter: the length around a shape
Regular Polygons: All sides and angles are equal

Quadrilateral Facts

Square
All sides equal size
All angles 90°
Opposite sides are parallel

Rectangle
All angles 90°
Opposite sides are parallel

Rhombus
All sides equal size
Opposite angles are equal



Parallelogram

Opposite sides are parallel
Opposite angles are equal
Co-interior angles



Kite

No parallel lines
Equal lengths on top sides
Equal lengths on bottom sides
One pair of equal angles

YEAR 9 — REASONING WITH GEOMETRY... Rotation & Translation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Identify the order of rotational symmetry
- Rotate a shape about a point on the shape
- Rotate a shape about a point not on a shape
- Translate by a given vector
- Compare rotations and reflections

Keywords

Rotate: a rotation is a circular movement

Symmetry: when two or more parts are identical after a transformation

Regular: a regular shape has angles and sides of equal lengths

Invariant: a point that does not move after a transformation

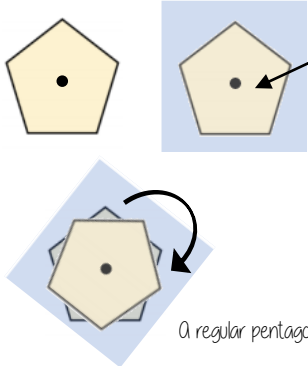
Vertex: a point two edges meet

Horizontal: from side to side

Vertical: from up to down

Rotational Symmetry

Tracing paper helps check rotational symmetry



1 Trace your shape (mark the centre point)

2 Rotate your tracing paper on top of the original through 360°

3 Count the times it fits back into itself

A regular pentagon has rotational symmetry of order 5

Translation and vector notation

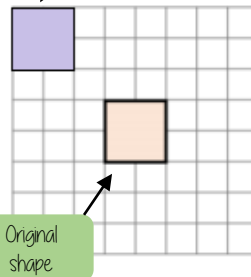
Vector Notation

$$\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

How far left or right to move
Negative value (left)
Positive value (right)

How far up or down to move
Negative value (down)
Positive value (up)

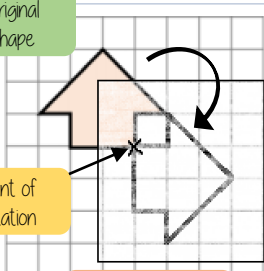
Translation $\begin{pmatrix} -3 \\ 3 \end{pmatrix}$



Every vertex has been translated by the same amount

Rotate from a point (in a shape)

Original shape



Point of rotation

Image 90° clockwise

1 Trace the original shape (mark the point of rotation)

2 Keep the point in the same place and turn the tracing paper

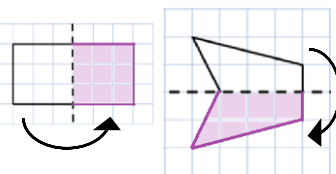
3 Draw the new shape



Clockwise

Anti-Clockwise

Compare rotations and reflections



R

Reflections are a mirror image of the original shape

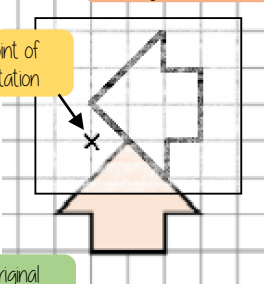
Information needed to perform a reflection:

- Line of reflection (Mirror line)

Rotate from a point (outside a shape)

Image 90° anti-clockwise

Point of rotation



Original shape

1 Trace the original shape (mark the point of rotation)

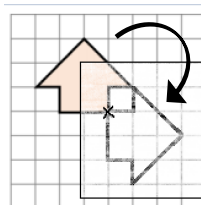
2 Keep the point in the same place and turn the tracing paper

3 Draw the new shape

Rotations are the movement of a shape in a circular motion

Information needed to perform a rotation:

- Point of rotation
- Direction of rotation
- Degrees of rotation



YEAR 9 — REASONING WITH GEOMETRY... Pythagoras' theorem

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Use square and cube roots
- Identify the hypotenuse
- Calculate the hypotenuse
- Find a missing side in a Right angled triangle
- Use Pythagoras' theorem on axes
- Explore proofs of Pythagoras' theorem

Keywords

Square number: the output of a number multiplied by itself

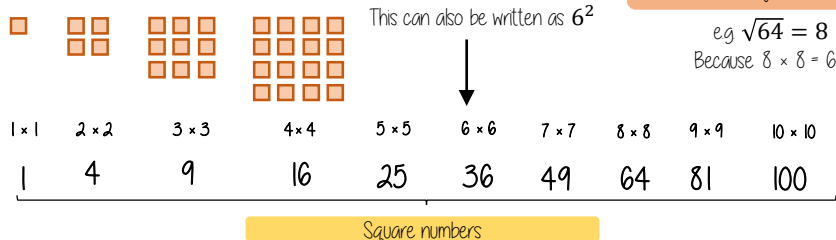
Square root: a value that can be multiplied by itself to give a square number

Hypotenuse: the largest side on a right angled triangle. Always opposite the right angle.

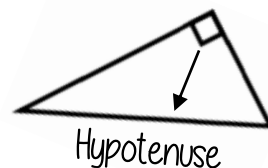
Opposite: the side opposite the angle of interest

Adjacent: the side next to the angle of interest

Squares and square roots

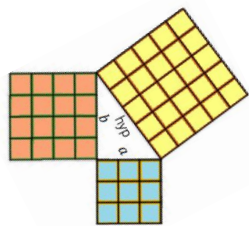


Identify the hypotenuse



The hypotenuse is always the longest side on a triangle because it is opposite the biggest angle.

Determine if a triangle is right-angled



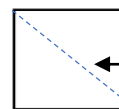
If a triangle is right-angled, the sum of the squares of the shorter sides will equal the square of the hypotenuse.

$$a^2 + b^2 = \text{hypotenuse}^2$$

eg $a^2 + b^2 = \text{hypotenuse}^2$

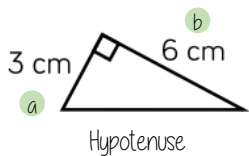
$$\begin{aligned} 3^2 + 4^2 &= 5^2 \\ 9 + 16 &= 25 \end{aligned}$$

Substituting the numbers into the theorem shows that this is a right-angled triangle



Polygons can still have a hypotenuse if it is split up into triangles and opposite a right angle

Calculate the hypotenuse



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

1 Substitute in the values for a and b

$$3^2 + 6^2 = \text{hypotenuse}^2$$

$$9 + 36 = \text{hypotenuse}^2$$

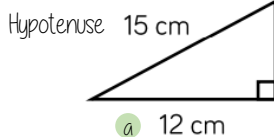
$$45 = \text{hypotenuse}^2$$

2 To find the hypotenuse square root the sum of the squares of the shorter sides

$$\sqrt{45} = \text{hypotenuse}$$

$$6.71\text{cm} = \text{hypotenuse}$$

Calculate missing sides



Either of the short sides can be labelled a or b

$$a^2 + b^2 = \text{hypotenuse}^2$$

$$12^2 + b^2 = 15^2$$

1 Substitute in the values you are given

$$144 + b^2 = 225$$

Rearrange the equation by subtracting the shorter square from the hypotenuse squared

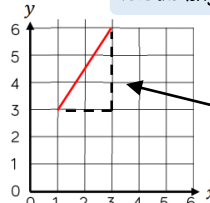
Square root to find the length of the side

$$b^2 = 111$$

$$b = \sqrt{111} = 10.54\text{ cm}$$

Pythagoras' theorem on a coordinate axis

Find the length of the line segment



The segment can be made into a right-angled triangle by adding the sides on the diagram

The line segment is the hypotenuse

$$a^2 + b^2 = \text{hypotenuse}^2$$

The lengths of a and b are the sides of the triangle

Be careful to check the scale on the axes

YEAR 9
KNOWLEDGE ORGANISERS



Summer 1: Reasoning with Proportion

12. Enlargement and Similarity

13. Solving Ratio and

Proportion Problems

14. Rates

Year 9

Summer 1: Reasoning with Proportion

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR	
Block 1 Enlargement & Similarity	Recognise enlargement and similarity	432	
	Enlarge a shape by a positive integer scale factor	433	
	Enlarge a shape by a positive integer scale factor from a point	434	
	Enlarge a shape by a positive fractional scale factor	435	
	Enlarge a shape by a negative scale factor (H)	436	
	Work out missing sides and angles in a pair of given similar shapes	437	
	Solve problems with similar triangles (H)	438	
	Explore ratios in right-angled triangles (H)	439	
Additional Key Vocabulary:			
Block 2 Solving ratio & proportion problems	Solve problems with direct proportion	440	
	Direct proportion and conversion graphs	441	
	Solve problems with inverse proportion	442	
	Graphs of inverse relationships (H)	443	
	Solve ratio problems given the whole or a part	444	
	Solve ' best buy ' problems	445	
	Solve problems ratio and algebra (H)	446	
Additional Key Vocabulary:			
Block 3 Rates	Solve speed, distance and time problems without a calculator	447	
	Solve speed, distance and time problems with a calculator	448	
	use distance/time graphs	449	
	Solve problems with density, mass, and volume	450	
	Solve flow problems and their graphs	451	
	Rates of change units	452	
	Convert compound units (H)	453	
Additional Key Vocabulary:			

YEAR 9

— SIMILARITY...

Congruence, similarity & enlargement

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Enlarge by a positive scale factor
- Enlarge by a fractional scale factor
- Identify similar shapes
- Work out missing sides and angles in similar shapes
- Use parallel lines to find missing angles
- Understand similarity and congruence

Keywords

Enlarge: to make a shape bigger (or smaller) by a given multiplier (scale factor)

Scale Factor: the multiplier of enlargement

Centre of enlargement: the point the shape is enlarged from

Similar: when one shape can become another with a reflection, rotation, enlargement or translation

Congruent: the same size and shape

Corresponding: items that appear in the same place in two similar situations

Parallel: straight lines that never meet (equal gradients)

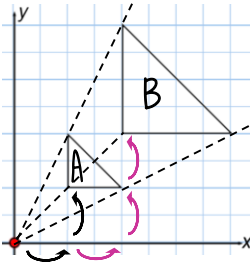
Positive scale factors R

Enlargement from a point

Enlarge shape A by SF 2 from (0,0)

The shape is enlarged by 2

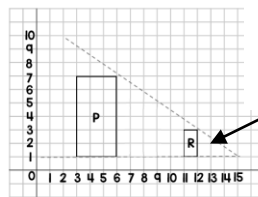
The distance from the point enlarges by 2



Fractional scale factors R

Fractions less than 1 make a shape **SMALLER**

R is an enlargement of P by a scale factor $\frac{1}{3}$ from centre of enlargement (15,1)



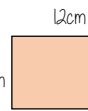
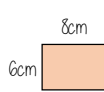
SF: $\frac{1}{3}$ - R is three times smaller than P

Identify similar shapes



Angles in similar shapes do not change.
e.g. if a triangle gets bigger the angles can not go above 180°

Similar shapes



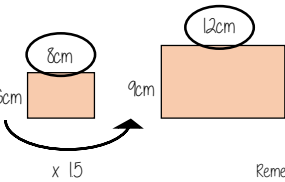
Scale Factor:
Both sides on the bigger shape are 1.5 times bigger

Compare sides: $6 : 9$
 $2 : 3$

$8 : 12$
 $2 : 3$

Both sets of sides are in the same ratio

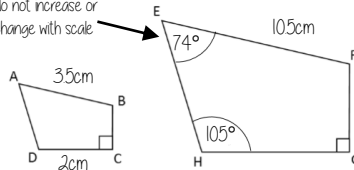
Information in similar shapes



Compare the equivalent side on both shapes

Scale Factor is the multiplicative relationship between the two lengths

Remember angles do not increase or change with scale



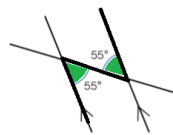
Shape ABCD and EFGH are similar

Notation helps us find the corresponding sides

AB and EF are corresponding

Angles in parallel lines R

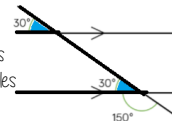
Alternate angles



Because alternate angles are equal the highlighted angles are the same size

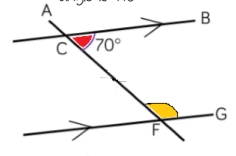
Corresponding angles

Because corresponding angles are equal the highlighted angles are the same size



Co-interior angles

Because co-interior angles have a sum of 180° the highlighted angle is 110°



Os angles on a line add up to 180° co-interior angles can also be calculated from applying alternate/ corresponding rules first

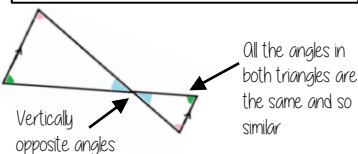
Similar triangles

Shares a vertex

Because corresponding angles are equal the highlighted angles are the same size

Parallel lines - all angles will be the same in both triangle

As all angles are the same this is similar - it only one pair of sides are needed to show equality



All the angles in both triangles are the same and so similar

YEAR 9 — REASONING WITH GEOMETRY...

Solving ratio & proportion problems

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve problems with direct proportion
- Use conversion graphs
- Solve problems with inverse proportion
- Solve ratio problems
- Solve 'best buy' problems

Keywords

Proportion: a comparison between two numbers

Ratio: a ratio shows the relative size of two variables

Direct proportion: as one variable is multiplied by a scale factor the other variable is multiplied by the same scale factor.

Inverse proportion: as one variable is multiplied by a scale factor the other is divided by the same scale factor.

Direct Proportion

As one variable changes the other changes at the same rate.

R



4 cans of pop = £2.40

4 cans of pop = £2.40
 $\times 0.5$ → 2 cans of pop = £1.20
 $\times 50$ → 200 cans of pop = £120

This multiplier is the same in the same way that this would be for ratio

This is a multiplicative change

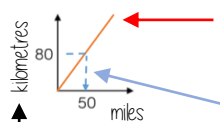
4 cans of pop = £2.40
 $\times 3$ → 12 cans of pop = £7.20

Sometimes this is easiest if you work out how much one unit is worth first
 e.g. 1 can of pop = £0.60

Conversion Graphs

Compare two variables

R



This is always a straight line because as one variable increases so does the other at the same rate

To make conversions between units you need to find the point to compare — then find the associated point by using your graph
 Using a ruler helps for accuracy
 Showing your conversion lines help as a "check" for solutions

Inverse Proportion

As one variable is multiplied by a scale factor the other is divided by the same scale factor

Examples of inversely proportional relationships

Time taken to fill a pool and the number of taps running

Time taken to paint a room and the number of workers

T is inversely proportional to G. When T=2 then G=20

T	1	2	8
G	40	20	5

$\div 2$ (from 1 to 2) $\times 4$ (from 2 to 8)
 $\times 2$ (from 40 to 20) $\div 4$ (from 20 to 5)

Best Buys

Have a directly proportional relationship

To calculate best buys you need to be able to compare the cost of one unit or units of equal amounts



Shop A

4 cans for £1.20

↓ $\pounds 1.20 \div 4$

Cost per item

1 can is £0.30
Or 30p

Shop B

3 cans for 93p

↓ $\pounds 0.93 \div 3$

1 can is £0.31
Or 31p

Shop A is the best value as it is 1p cheaper per can of pop



Shop A

4 cans for £1.20

↓ $4 \div \pounds 1.20$

Cost per pound

£1 buys 3.333 cans of pop

3 cans for 93p

↓ $3 \div \pounds 0.93$

£1 buys 3.23 cans of pop

Shop A is still shown as being the best value but pay attention to the unit you are calculating, per item or per pound

Best value is the most product for the lowest price per unit

Sharing a whole into a given ratio

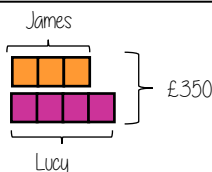
R

James and Lucy share £350 in the ratio 3:4.
Work out how much each person earns

Model the Question

James: Lucy

3 : 4



£350 ÷ 7 = £50

□ = one part = £50

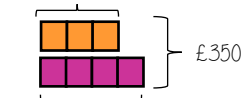
Find the value of one part

Whole: £350
7 parts to share between
(3 James, 4 Lucy)

Put back into the question

James: Lucy

James = 3 × £50 = £150



Lucy = 4 × £50 = £200

$\times 50$ 3 : 4 $\times 50$
 $\pounds 150 : \pounds 200$

Finding a value given 1:n (or n:1)

R

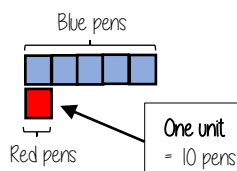
Inside a box are blue and red pens in the ratio 5:1
If there are 10 red pens how many blue pens are there?

Model the Question

Blue : Red

5 : 1

□ = one part = 10 pens

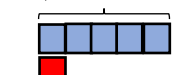


Put back into the question

Blue: Red

$\times 10$ 5 : 1 $\times 10$
 $\pounds 50 : 10$

Blue pens = 5 × 10 = 50 pens



Red pens = 1 × 10 = 10 pens

There are 50 Blue Pens



YEAR 9 — REASONING WITH GEOMETRY... Rates

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Solve speed, distance, time questions
- Use distance time graphs
- Solve density, mass, volume problems
- Solve flow problems
- Use flow graphs
- Interpret rates of change and their units

Keywords

Convert: change

Mass: a measure of how much matter is in an object. Commonly measured by weight

Origin: the coordinate (0, 0)

Volume: the amount of 3D space a shape takes up

Substitute: putting numbers where letters are — replacing numbers into a formula

Speed, Distance, Time

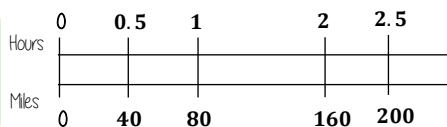
'per' for every

e.g. 80 miles per hour (mph)

Travel 80 miles every hour

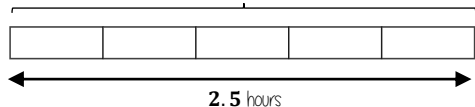
$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

You can use a double number line to help you calculate distance



e.g. A boat travels at a constant speed for 2.5 hours. It travels 300 miles.

300 miles



Bar models can help to calculate mph

Each part is half an hour
Each part is 60 miles

Speed, Distance, Time



Before calculations — make sure you are working in the same units as the speed



Learn or learn how to rearrange the formula for speed, distance and time

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

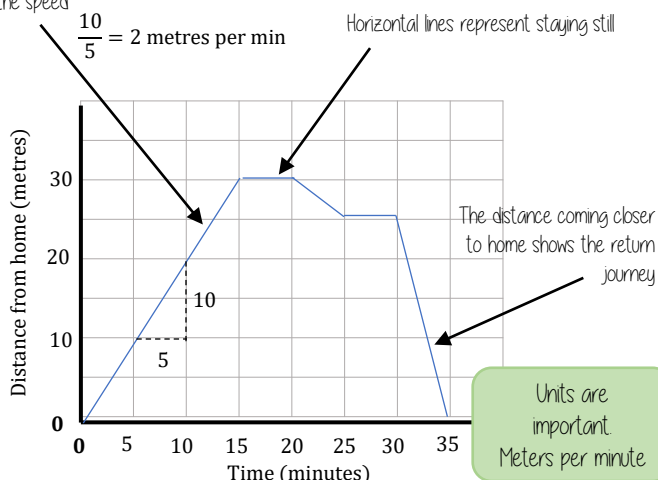
Substitute in the variables given

$$\text{distance} = \text{speed} \times \text{time}$$

Distance — Time graphs

The steeper a gradient the faster the speed

Gradient = speed

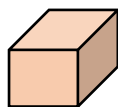


Density, Mass, Volume

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{volume} = \frac{\text{mass}}{\text{density}}$$

$$\text{mass} = \text{volume} \times \text{density}$$



volume of prism = Area of cross section \times Depth



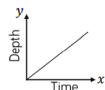
Flow problems & graphs



This will fill at a constant rate, then as the space decreases it will speed up and the neck of the bottle fill at a faster constant speed



The cylinder will fill at a constant speed



Units are important. Ensure any volume calculations are the same unit as the rate of flow

Rates of change & units

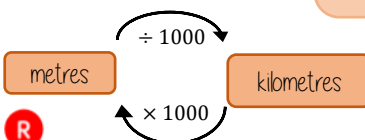
Common rates of change relationships

Revisit your conversions between units of length and capacity

Speed: miles per hour

Exchange rates: euros per pounds

Density: mass per volume



YEAR 9
KNOWLEDGE ORGANISERS



**Summer 2: Representations and
Revision**

15. Probability

16. Algebraic Representation

Year 9
Summer 2: Representations

Key Vocabulary Challenge

Use as much key vocabulary in your lessons as you can, both in your written and verbal answers. All Key vocabulary is in bold and underlined below.

		WR	
Block 4 Probability	Single <u>event probability</u>	354	
	Relative <u>frequency</u> - include <u>convergence</u>	355	
	Expected <u>outcomes</u>	356	
	<u>Independent events</u>	357	
	Use <u>tree diagrams (H)</u>	358	
	Use <u>tree diagrams</u> to solve 'without replacement' problems (H)	359	
	Use diagrams to work out <u>probabilities</u>	360	
Additional Key Vocabulary:			
Block 5 Algebraic representation	Draw and interpret <u>quadratic graphs</u>	361	
	Interpret graphs, including <u>reciprocal</u> and piece-wise	362	
	Investigate graphs of <u>simultaneous equations (H)</u>	363	
	Represent <u>inequalities</u>	364	
Additional Key Vocabulary:			

YEAR 9 — REPRESENTATIONS...

Probability

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Find single event probability
- Find relative frequency
- Find expected outcomes
- Find independent events
- Use diagrams to work out probabilities

Keywords

Probability: the chance that something will happen

Relative Frequency: how often something happens divided by the outcomes

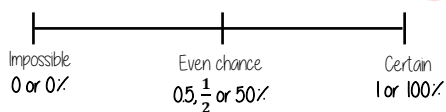
Independent: an event that is not effected by any other events.

Chance: the likelihood of a particular outcome.

Event: the outcome of a probability — a set of possible outcomes.

Biased: a built in error that makes all values wrong by a certain amount.

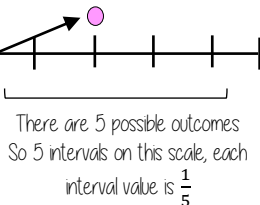
The probability scale



The more likely an event the further up the probability it will be in comparison to another event (It will have a probability closer to 1)



There are 2 pink and 2 yellow balls, so they have the same probability



R

Single event probability

Probability is always a value between 0 and 1



The probability of getting a blue ball is $\frac{1}{5}$
∴ The probability of NOT getting a blue ball is $\frac{4}{5}$

The sum of the probabilities is 1

The table shows the probability of selecting a type of chocolate

Dark	Milk	White
0.15	0.35	

$$P(\text{white chocolate}) = 1 - 0.15 - 0.35 = 0.5$$



R

Relative Frequency

$$\frac{\text{Frequency of event}}{\text{Total number of outcomes}}$$

Remember to calculate or identify the overall number of outcomes!

Colour	Frequency	Relative Frequency
Green	6	0.3
Yellow	12	0.6
Blue	2	0.1
	20	

Relative frequency can be used to find expected outcomes

e.g. Use the relative probability to find the expected outcome for green if there are 100 selections

$$\text{Relative frequency} \times \text{Number of times} \\ 0.3 \times 100 = 30$$

Expected outcomes

Expected outcomes are estimations. It is a long term average rather than a prediction.

Dark	Milk	White
0.15	0.35	0.5

The sum of the probabilities is 1

An experiment is carried out 400 times

Show that dark chocolate is expected to be selected 60 times

$$0.15 \times 400 = 60$$

Independent events



The rolling of one dice has no impact on the rolling of the other. The individual probabilities should be calculated separately.

$$\text{Probability of event 1} \times \text{Probability of event 2}$$



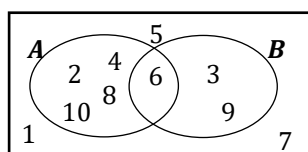
$$P(5) = \frac{1}{6} \quad P(R) = \frac{1}{4}$$

Find the probability of getting a 5 and a red

$$P(5 \text{ and } R) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

Using diagrams

Recap Venn diagrams, Sample space diagrams and Two-way tables



	Car	Bus	Wak	Total
Boys	15	24	14	53
Girls	6	20	21	47
Total	21	44	35	100

The possible outcomes from tossing a coin

The possible outcomes from rolling a dice

	1	2	3	4	5	6
H	1H	2H	3H	4H	5H	6H
T	1T	2T	3T	4T	5T	6T

YEAR 9 — REPRESENTATIONS...

Algebraic Representation

@whisto_maths

What do I need to be able to do?

By the end of this unit you should be able to:

- Draw quadratic graphs
- Interpret quadratic graphs
- Interpret other graphs including reciprocals
- Represent inequalities

Keywords

Quadratic: a curved graph with the highest power being 2. Square power.

Inequality: makes a non equal comparison between two numbers

Reciprocal: a reciprocal is 1 divided by the number

Cubic: a curved graph with the highest power being 3. Cubic power.

Origin: the coordinate (0, 0)

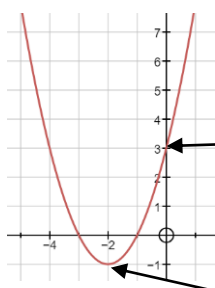
Parabola: a 'u' shaped curve that has mirror symmetry

Quadratic Graphs

$$y = x^2 + 4x + 3$$

If x^2 is the highest power in your equation then you have a quadratic graph

It will have a parabola shape



Substitute the x values into the equation of your line to find the y coordinates

x	-4	-3	-2	-1	0	1
y	3	0	-1	0	3	8

Coordinate pairs for plotting (-3, 0)

Plot all of the coordinate pairs and join the points with a curve (freehand)

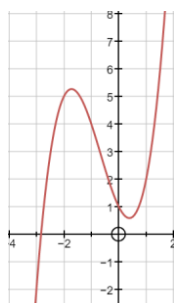
Quadratic graphs are always symmetrical with the turning point in the middle

Interpret other graphs

Cubic Graphs

$$y = x^3 + 2x^2 - 2x + 1$$

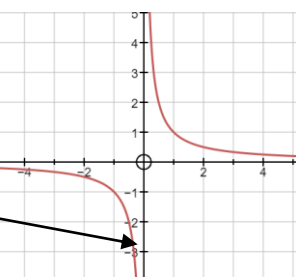
If x^3 is the highest power in your equation then you have a cubic graph



Reciprocal Graphs

$$y = \frac{1}{x}$$

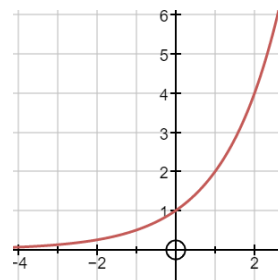
Reciprocal graphs never touch the y axis
This is because x cannot be 0
This is an asymptote



Exponential Graphs

$$y = 2^x$$

Exponential graphs have a power of x

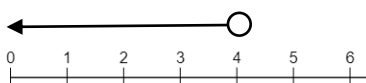


Represent Inequalities

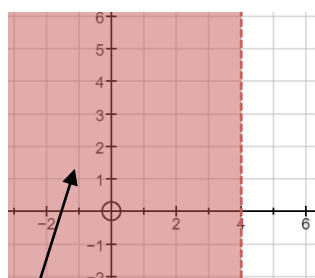
Multiple methods of representing inequalities

$$x < 4$$

All values are less than 4



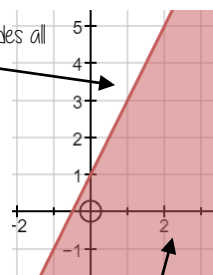
The shaded area indicates all possible values of x



The dotted line shows that the inequality does not include these points

The solid line shows that the inequality includes all the points on this line

$$y \geq 2x + 1$$



The shaded area indicates all possible solutions to this inequality